2.1.12. Suppos a, b are both zero, then

$b=0 \Rightarrow a+b=a$

$a=0 \Rightarrow a+b=b$

thus $a=b$.

2.1.13 (a) $(0,0)$ is the zero vector

$(v,\hat{v})+(w,\hat{w})=(v+w, \hat{v}+\hat{w}) \in V\times W$ since

$v+w \in V$ & $\hat{v}+\hat{w} \in W$

$c(v,w)=(cv,cw) \in V\times W$ since $cv \in V$ $cw \in W$

(b) the map $T: \mathbb{R}\times\mathbb{R} \to \mathbb{R}^2$

$(a,b) \mapsto ae_1+be_2$

is an isomorphism (show that it's invertible).

(c) the map $T: \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^{m+n}$

$((a_1 \cdots a_m),(b_1 \cdots b_n)) \mapsto (a_1 \cdots a_m b_1 \cdots b_n)$

is an isomorphism.

2.2.29 a) i) $v - v = 0 \in W \Rightarrow v \sim_W v \quad \forall\, v \in V$

ii) $u - v \in W \Leftrightarrow -(u-v) = v - u \in W$

thus $u \sim_W v \Leftrightarrow v \sim_W u$

iii) $u \sim_W v \;\&\; v \sim_W z$

$\Rightarrow u - v \in W \quad v - z \in W$

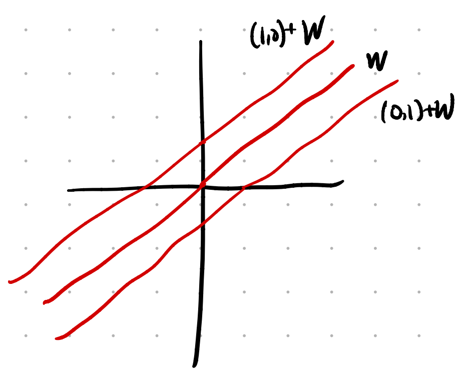
$\Rightarrow (u-v) + (v-z) \in W$

$\Leftrightarrow u - z \in W \Rightarrow u \sim_W z.$

b) $[0]_W = \{ v \in V \mid v \sim_W 0 \} = \{ v \in V \mid v - 0 \in W \}$

$= \{ v \in V \mid v \in W \} = W$

c)



d) $[v]_W = \{ w + v \mid w \in W \}$ is affine subspace by def.

e) See lecture

2.3.18 $v_{m+1} \cdots v_n$ are linear comb of $v_1 \cdots v_m$

So a linear comb of $v_1 \cdots v_n$ is also a linear comb of $v_1 \cdots v_m$, thus

$\text{span}(v_1 \cdots v_n) = \text{span}(v_1 \cdots v_m) = V$.

2.4.22 See lecture (lemma 2.19).

$\dim V = n$ when $v_1 \cdots v_n$ linear ind.

2.4.27 See lecture (prop 3.33)